ASSERTION AND REASON

Some questions (Assertion–Reason type) are given below. Each question contains **Statement – 1 (Assertion)** and **Statement – 2 (Reason)**. Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct. So select the correct choice :

Choices are :

(A) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is** a correct explanation for **Statement – 1**.
(B) **Statement – 1** is True, **Statement – 2** is True; **Statement – 2 is NOT** a correct explanation for **Statement – 1**.
(C) **Statement – 1** is True, **Statement – 2** is False.
(D) **Statement – 1** is False, **Statement – 2** is True.

INDEFINITE & DEFINITE INGEGRATION

129. Let F(x) be an indefinite integral of $\cos^2 x$.
Statement-1: The function F(x) satisfies $F(x + \pi) = F(x)\ \forall$ real x
Statement-2: $\cos^2(x + \pi) = \cos^2 x$.

130. **Statement-1:** $\int |x|\,dx$ can not be found while $\int_{-1}^{1} |x|\,dx$ can be found.
Statement-2: |x| is not differentiable at x = 0.

131. **Statement-1:** $\int \left(\frac{1}{1+x^4}\right) dx = \tan^{-1}(x^2) + C$ **Statement-2:** $\int \frac{1}{1+x^2} dx = \tan^{-1}x + C$

132. **Statement-1:** If y is a function of x such that $y(x-y)^2 = x$ then $\int \frac{dx}{x-3y} = \frac{1}{2}\left[\log(x-y)^2 - 1\right]$
Statement-2: $\int \frac{dx}{x-3y} = \log(x-3y) + c$

133. **Statement–1 :** $f(x) = \log\sec x - \frac{x^2}{2}$ **Statement–2 :** f(x) is periodic

134. **Statement–1 :** $\int \frac{x^{9/2}}{\sqrt{1+x^{11}}} dx = \frac{2}{11} \ln\left|x^{11/2} + \sqrt{1+x^{11}}\right| + c$
Statement–2 : $\int \frac{dx}{\sqrt{1+x^2}} = \ln|x + \sqrt{1+x^2}| + c$

135. **Statement–1 :** $\int_0^{10} \left[\tan^{-1} x\right] dx = 10 - \tan 1$; where [x] = G.I.F.
Statement–2 : $[\tan^{-1} x] = 0$ for $0 < x < \tan 1$ and $[\tan^{-1} x] = 1$ for $\tan 1 \le x < 10$.

136. **Statement–1 :** $\int_0^{\pi/2} \frac{dx}{1+\tan^3 x} = \frac{\pi}{4}$
Statement–2 : $\int_0^a f(x)\,dx = \int_0^a f(a+x)\,dx$

$$\int_0^{\pi/2} \frac{dx}{1+\tan^3 x} = \int_0^{\pi/2} \frac{dx}{1+\cot^3 x} = \frac{\pi}{4} \qquad \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx.$$

137. **Statement–1 :** $\int_0^{\pi} \sqrt{1-\sin^2 x}\,dx = 0$ **Statement–2 :** $\int_0^{\pi} \cos x\,dx = 0$.

138. **Statement–1 :** $\int e^x\left(\tan x + \sec^2 x\right) dx = e^x \tan x + c$

Statement–2 : $\int e^x (f(x)+f'(x))\,dx = e^x f(x)+c$.

139. **Statement–1 :** If f(x) satisfies the conditions of Rolle's theorem in [α, β], then $\int_{\alpha}^{\beta} f'(x)\,dx = \beta - \alpha$

Statement–2 : If f(x) satisfies the conditions of Rolle's theorem in [α, β], then $\int_{\alpha}^{\beta} f'(x)\,dx = 0$

140. **Statement–1 :** $\int_0^{4\pi} [|\sin x|+|\cos x|]dx$, where [·] denotes G.I.F. equals 8π.

Statement–2 : If f(x) = |sinx| + |cosx|, then $1 \le f(x) \le \sqrt{2}$.

141. Let f(x) be a continuous function such that $\int_n^{n+1} f(x)\,dx = n^3,\ n \in I$

Statement–1 : $\int_{-3}^{3} f(x)\,dx = 27$ **Statement–2 :** $\int_{-2}^{2} f(x)\,dx = 27$

142. Let $I_n = \int_1^e (\ell n x)^n\,dx,\ n \in N$

Statement–I : $I_1, I_2, I_3 \ldots$ is an increasing sequence.
Statement–II : $\ell n\, x$ is an increasing function.

143. Let f be a periodic function of period 2. Let $g(x) = \int_0^x f(t)\,dt$ and h(x) = g(x + 2) – g(x).

Statement–1 : h is a periodic function. **Statement–2** : g(x + 2) – g(x) = g(2).

144. **Statement–1** : $\int \frac{e^x}{x}(1+x\log x)\,dx = e^x \log x + c$

Statement–2 : $\int e^x (f(x)+f'(x))\,dx = e^x f(x)+c$.

145. **Statement–1** : If $I_1 = \int_x^1 \frac{dt}{1+t^2}$ and $I_2 \int_1^{1/x} \frac{dt}{1+t^2},\ x>0$ then $I_1 = I_2$.

Statement–2 : $\int_{-2}^{2} \min.\{x-[x],\ -x-[-x]\}\,dx = 0$

146. **Statement–1** : $8 < \int_4^6 2x\,dx < 12$.

Statement–2 : If m is the smallest and M is the greatest vlaue of a function f(x) in an interval (a, b), then the vlaue of the integral $\int_a^b f(x)dx$ is such that for a < b, we have $M(b-a) \le \int_a^b f(x)\,dx \le M(b-a)$.

147. **Statement–1 :** $\int e^{ax}\sin bx dx = \frac{e^{ax}}{A}$(asinbx – bcosbx)+c Then A is $\sqrt{a^2+b^2}$

Statement–2 : $\int e^x\left(\frac{1+\sin x\cos x}{\cos^2 x}\right)dx = e^x \tan x + c$

148. **Statement–1 :** $\int \frac{d(x^2+1)}{\sqrt{\lambda^2+2}}$ is equal to $2\sqrt{x^2+2}+c$

Statement–2 : $\int \frac{x^{a/2}}{\sqrt{1+x^{11}}}dx$ is 2/11 ln $|x + \sqrt{1+x^{11}}| + c$

149. **Statement–1 :** $\int_{\pi/6}^{\pi/3} \frac{1}{1+\tan^3 x}$ is $\pi/12$ **Statement–2 :** $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

150. **Statement–1 :** If f satisfies $f(x + y) = f(x) + f(y)\ \forall\ x,\ y \in R$ then $\int_{-5}^{5} f(x)\,dx = 0$

Statement–2 : If f is an odd function then $\int_{-a}^{a} f(x)\,dx = 0$

151. **Statement–1 :** If f(x) is an odd function of x then $\int_a^x f(t)\,dt$ is an even function of (n)

Statement–2 : If graph of y = f(x) is symmetric about y–axis then f(x) is always an even function.

152. **Statement–1 :** Area bounded by y = {x}, {x} is fractional part of x = 0, x = 2 and x–axis is 1.

Statement–2 : Area bounded by y = |sinx|, x = 0, x = 2π is 2 sq. unit.

153. **Statement-1:** $\lim\limits_{n\to\infty}\left(\frac{1}{\sqrt{4n^2-1}}+\frac{1}{\sqrt{4n^2-2^2}}+\ldots+\frac{1}{\sqrt{3}n}\right)=\frac{\pi}{3}$

Statement-2: $\lim\limits_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)=\int_0^1 f(x)\,dx$, symbols have their usual meaning.

154. **Statement-1:** If $I_n = \int \tan^n x\,dx$, then $5(I_4 + I_6) = \tan^5 x$.

Statement-2: If $I_n = \int \tan^4 x\,dx$, then $\frac{\tan^{n-1} x}{n} - I_{n-2} = I_n$, $n \in N$.

155. **Statement-1:** If a > 0 and $b^2 - 4ac < 0$, then the value of the integral $\int \frac{dx}{ax^2+bx+c}$ will be of the type $\mu \tan^{-1}\left(\frac{x+A}{B}\right)+c$, where A, B, C, μ are constants.

Statement-2: If a > 0, $b^2 - 4ac < 0$ then $ax^2 + bx + c$ can be written as sum of two squares.

156. **Statements-1:** $\int \frac{x^2-x+1}{(x^2+1)^{3/2}}e^x\,dx = \frac{e^x}{\sqrt{x^2+1}}+c$ **Statements-2:** $\int e^x\,(f(x)+f'(x)\,dx = e^x f(x) + c$

157. **Statements-1:** $\int \frac{x^2-2}{(x^4+5x^2+4)\tan^{-1}\left(\frac{x^2+2}{x}\right)}dx$ = log $|\tan^{-1}(x + 2/x)| + c$

Statements-2: $\int \frac{dx}{a^2+x^2}=\frac{1}{a}\tan^{-1}\frac{x}{a}+c$

158. **Statements-1:** $\int \frac{\ln\frac{x}{e}}{(\ln x)^2}=\frac{x}{\ln x}+c$ **Statements-2:** $\int e^x\,(f(x) + f'(x))\,dx = e^x f(x) + c.$

159. **Statements-1:** $\int \frac{1}{x^3\sqrt{1+x^4}}dx=-\frac{1}{2}\sqrt{1+\frac{1}{x^4}}+c$ **Statements-2:** For integration by parts we have to follow ILATE rule.

160. **Statements-1:** A function F(x) is an antiderivative of a function f(x) if $F'(x) = f(x)$

Statements-2: The functions $x^2 + 1$, $x^2 - \pi$, $x^2 + \sqrt{2}$ are all antiderivatives of the function 2x.

161. **Statements-1:** $\int_a^b \frac{|x|}{x}\,dx = |b| - |a|$, $a < b$

Statements-2: If f(x) is a function continuous every where in the interval (a, b) except x = c then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

162. **Statements-1:** $4 \le \int_1^3 \sqrt{3+x^3}dx \le 2\sqrt{30}$

Statements-2: m and M be the least and the maximum value of a continuous function

y = f(x) in [a, b] then $m(b-a) \le \int_a^b f(x)dx \le M(b-a)$

163. **Statements-1:** $1 < \int_0^1 e^{x^2}dx < e$

Statements-2: if f(x) ≤ g(x) ≤ h(x) in (a, b) then $\int_a^b f(x)dx \le \int_a^b g(x)dx \le \int_a^b h(x)dx$

164. **Statements-1:** $\left|\int_0^1 \sqrt{1+x^4}dx\right| < \sqrt{1.2}$

Statements-2: For any functions f(x) and g(x), integrable on the interval (a,b), then

$\left|\int_a^b f(x)g(x)dx\right| \le \sqrt{\int_a^b f^2(x)dx \int_a^b g^2(x)dx}$

165. **Statements-1:** $\int_{-1}^1 \frac{1}{x^2}dx = -2$

Statements-2: If F(x) is antiderivative of a continuous function (a, b) then $\int_a^b f(x)dx = F(b)-F(a)$

166. **Statements-1:** $\frac{\cos x}{(1+\sin x)^2}$ can be integrated by substitution it sinx = t.

Statements-2: All integrands are integrated by the method of substitution only.

167. **Statement-1 :** $\int e^x\left(\frac{1+\sin x \cos}{\cos^2 x}\right)dx = \int e^x \tan x + c$

Statement-2 : $\int e^x (f(x) + f'(x))dx = e^x f'(x) + c$

168. **Statements-1:** $\int e^x(x+1)\cos^2(x.e^x)dx = \frac{1}{2}x.e^x + \frac{1}{4}\sin 2(x.e^x) + C$

Statements-2: $\int f(\phi(x))\phi'(x)dx, \{\phi(x) = t\}$ equals $\int f(t)dt$.

169. **Statements-1:** $\int \log x dx = x\log x - x + c$

Statements-2: $\int uvdx = u\int vdx + \int\left(\frac{du}{dx}\int vdx\right)dx$

170. **Statements-1:** $\int e^x\left(\frac{x^2+4x+2}{x^2+4x+4}\right)dx = \frac{e^x}{(x+2)^2} + C$ **Statements-2:** $\int e^x(f(x)+f'(x))dx = e^x f(x) + C$

171. **Statements-1:** $\int_{-1}^1 \frac{\sin x - x^2}{3+|x|} = -2\int_0^1 \frac{x^2}{3+|x|}$ **Statements-2:** $\int_{-a}^a f(x) = dx = \int_0^a f(x)dx + \int_0^a f(-x)dx$

172. **Statements-1:** The value of $\int_0^1 \sqrt{(1+x)(1+x^3)}dx$ can not exceed $\sqrt{\frac{15}{8}}$

Statements-2: If $m \le f(x) \le M \ \forall \ x \in [a, b]$ then $m(b-a) \le \int_a^b f(x)dx \le (b-a)M$

173. **Statements-1:** $\int_0^{\pi/2} \frac{(\sin x)^{5/2}}{(\sin x)^{5/2} + (\cos x)^{5/2}} dx = \frac{\pi}{4}$ **Statements-2:** Area bounded by $y = 3x$ and $y = x^2$ is $= \frac{9}{2}$ sq. units

174. **Statements-1:** $\int \frac{10x^9 + 10^x \log_e 10}{10^x + x^{10}} dx = \log|10^x + x^{10}| + c$ **Statements-2:** $\int \frac{f'(x)}{f(x)} dx = \log|f(x)| + c$

175. **Statements-1:** $\int \frac{e^x(1+x)}{\cos^2(xe^x)} dx = \tan(xe^x) + c$ **Statements-2:** $\int \sec^2 x dx = \tan x + c$

176. **Statement-1 :** $f(x) = \int_1^x \frac{\ln t \, dt}{1+t+t^2} (x > 0)$, then $f(x) = -f\left(\frac{1}{x}\right)$

Statements-2: $f(x) = \int_1^x \frac{\ln t \, dt}{t+1}$, then $f(x) + f\left(\frac{1}{x}\right) = \frac{1}{2} (\ln x)^2$

177. **Statement-1 :** $\int_{-1}^{1} \frac{\sin x - x^2}{3-|x|} dx = \int_0^1 \frac{-2x^2}{3-|x|} dx$.

Statements-2: Since $\frac{\sin x}{3-|x|}$ is an odd function. So, that $\int_{-1}^{1} \frac{\sin x}{3-|x|} = 0$.

178. **Statements-1 :** $\int_0^{n\pi+t} |\sin x| dx = (2n+1) - \cos t \ (0 \le t \le \pi)$

Statements-2: $\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ and $\int_0^{na} f(x) dx = n \int_0^a f(x) dx$ if $f(a + x) = f(x)$

179. **Statements-1:** The value of the integral $\int_0^1 e^{x^2} dx$ belongs to [0, 1]

Statements-2: If m & M are the lower bound and the upper bounds of f(x) over [a, b] and f is integrable, then $m(b-a) \le \int_a^b f(x) dx \le M(b-a)$.

180. **Statements-1:** $\int_0^{\infty} [\cot^{-1} x] dx = \cot 1$, where $[\cdot]$ denotes greatest integer function.

Statements-2: $\int_a^b f(x) dx$ is defined only if f(x) is continuous in (a, b) $[\cdot]$ function is discontinuous at all integers

181. **Statements-1:** $\int_{-4}^{4} \left(\sqrt{1+x+x^2} - \sqrt{1-x+x^2}\right) dx = 0$ **Statements-2:** $\int_{-a}^{a} f(x)dx = 0$ if f(x) is an odd function.

182. **Statements-1:** All continuous functions are integrable
Statements-2: If a function $y = f(x)$ is continuous on an interval [a,b] then its definite integral over [a, b] exists.

183. **Statements-1:** If f(x) is continuous on [a, b], $a \ne b$ and if $\int_a^b f(x) dx = 0$, then f(x) = 0 at least once in [a, b]

Statements-2: If f is continuous on [a, b], then at some point c in [a, b] $f(c) = \frac{1}{b-a} \int_a^b f(x) dx$

184. **Statements-1:** $\int_{-4}^{4} |x+2| dx = 50$ **Statements-2:** $\int_0^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx$ where $C \in (A, B)$

185. **Statements-1:** $\int_{-2}^{2}\log\left(\frac{1+x}{1-x}\right)dx = 0$ **Statements-2:** If f is an odd function $\int_{-a}^{a} f(x)\,dx = 0$

186. **Statement-1** If $\int_0^\infty e^{-ax}dx = \frac{1}{a}$ then $\int_0^\infty x^m e^{-ax}dx = \frac{m!}{a^{m+1}}$ **Statement-2 :** $\frac{d^n}{dx^n}(e^{kx}) = k^n e^{kx}$ and $\frac{d^n}{dx^n}\left(\frac{1}{x}\right) = \frac{(-1)^n n!}{x^{n+1}}$

187. **Statement-1 :** $\int_0^{10}\{x-[x]dx = 5$ **Statements-2:** $\int_a^{na} f(x)dx = n\int_0^a f(x)\,dx$

188. **Statements-1:** $\int_0^\pi |\cos x|\,dx = 2$ **Statements-2:** $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$ where a < c < b.

189. **Statements-1:** $\int_0^\pi \frac{e^{\cos x}}{e^{\cos x}+e^{-\cos x}}dx = \pi$ **Statements-2:** $\int_a^b f(x)dx = \int_a^b f(a+b-x)dx$

190. **Statements-1:** $\int_0^{1000} e^{x-[x]}dx = 1000(e-1)$ **Statements-2:** $\int_0^n e^{x-[x]}dx = n\int_0^1 e^{x-[x]}dx$

191. **Statements-1:** $\int_0^\pi \frac{dx}{1+2^{\tan x}} = \frac{\pi}{2}$ **Statements-2:** $\int_a^b f(x)\,dx = \int_a^b f(a+b-x)\,dx$

ANSWER

129. D	130. B	131. D	132. C	133. A	134. A	135. A	136. C
137. D	138. A	139. A	140. D	141. D	142. D	143. A	144. A
145. C	146. A	147. D	148. C	149. A	150. A	151. C	152. C
153. D	154. C	155. A	156. C	157. A	158. A	159. B	160. B
161. A	162. A	163. A	164. A	165. D	166. C	167. C	168. A
169. C	170. A	171. A	172. A	173. B	174. A	175. A	176. D
177. A	178. A	179. D	180. A	181. A	182. B	183. A	184. A
185. A	186. A	187. C	188. A	189. D	190. A	191. A	

Que. from Compt. Exams

(Indefinite Integral)

1. $\int \frac{dx}{\cos(x-a)\cos(x-b)} =$

(a) $\text{cosec}\ (a-b)\log\frac{\sin(x-a)}{\sin(x-b)}+c$ (b) $\text{cosec}(a-b)\log\frac{\cos(x-a)}{\cos(x-b)}+c$

(c) $\text{cosec}(a-b)\log\frac{\sin(x-b)}{\sin(x-a)}+c$ (d) $\text{cosec}(a-b)\log\frac{\cos(x-b)}{\cos(x-a)}+c$

2. $\int \frac{dx}{\sqrt{x+a}+\sqrt{x+b}} =$ **[AISSE 1989]**

(a) $\frac{2}{3(b-a)}[(x+a)^{3/2}-(x+b)^{3/2}]+c$ (b) $\frac{2}{3(a-b)}[(x+a)^{3/2}-(x+b)^{3/2}]+c$

(c) $\frac{2}{3(a-b)}[(x+a)^{3/2}+(x+b)^{3/2}]+c$ (d) None of these

3. $\int \frac{3\cos x+3\sin x}{4\sin x+5\cos x}dx =$ **[EAMCET 1991]**

(a) $\frac{27}{41}x-\frac{3}{41}\log(4\sin x+5\cos x)$ (b) $\frac{27}{41}x+\frac{3}{41}\log(4\sin x+5\cos x)$

(c) $\frac{27}{41}x-\frac{3}{41}\log(4\sin x-5\cos x)$ (d) None of these

4. If $\int(\sin 2x+\cos 2x)\,dx = \frac{1}{\sqrt{2}}\sin(2x-c)+a$, then the value of a and c is **[Roorkee 1978]**

(a) $c = \pi/4$ and $a = k$ (an arbitrary constant) (b) $c = -\pi/4$ and $a = \pi/2$
(c) $c = \pi/2$ and a is an arbitrary constant (d) None of these

5. $\int \frac{x^3 - x - 2}{(1-x^2)} dx =$ **[AI CBSE 1985]**

(a) $\log\left(\frac{x+1}{x-1}\right) - \frac{x^2}{2} + c$ (b) $\log\left(\frac{x-1}{x+1}\right) + \frac{x^2}{2} + c$ (c) $\log\left(\frac{x+1}{x-1}\right) + \frac{x^2}{2} + c$ (d) $\log\left(\frac{x-1}{x+1}\right) - \frac{x^2}{2} + c$

6. $\int \frac{\sin^8 x - \cos^8 x}{1 - 2\sin^2 x \cos^2 x} dx =$ **[IIT 1986]**

(a) $\sin 2x + c$ (b) $-\frac{1}{2}\sin 2x + c$ (c) $\frac{1}{2}\sin 2x + c$ (d) $-\sin 2x + c$

7. $\int \frac{x^2 dx}{(a+bx)^2} =$ **[IIT 1979]**

(a) $\frac{1}{b^2}\left[x + \frac{2a}{b}\log(a+bx) - \frac{a^2}{b}\frac{1}{a+bx}\right]$ (b) $\frac{1}{b^2}\left[x - \frac{2a}{b}\log(a+bx) + \frac{a^2}{b}\frac{1}{a+bx}\right]$

(c) $\frac{1}{b^2}\left[x + \frac{2a}{b}\log(a+bx) + \frac{a^2}{b}\frac{1}{a+bx}\right]$ (d) $\frac{1}{b^2}\left[x + \frac{a}{b} - \frac{2a}{b}\log(a+bx) - \frac{a^2}{b}\frac{1}{a+bx}\right]$

8. $\int \frac{dx}{(1+x^2)\sqrt{p^2 + q^2(\tan^{-1} x)^2}} =$

(a) $\frac{1}{q}\log[q\tan^{-1} x + \sqrt{p^2 + q^2(\tan^{-1} x)^2}] + c$ (b) $\log[q\tan^{-1} x + \sqrt{p^2 + q^2(\tan^{-1} x)^2}] + c$

(c) $\frac{2}{3q}(p^2 + q^2 \tan^{-1} x)^{3/2} + c$ (d) None of these

9. $\int \frac{x^5}{\sqrt{1+x^3}} dx =$ **[IIT 1985]**

(a) $\frac{2}{9}(1+x^3)^{3/2} + c$ (b) $\frac{2}{9}(1+x^3)^{3/2} + \frac{2}{3}(1+x^3)^{1/2} + c$

(c) $\frac{2}{9}(1+x^3)^{3/2} - \frac{2}{3}(1+x^3)^{1/2} + c$ (d) None of these

10. $\int \frac{dx}{\sin x - \cos x + \sqrt{2}}$ equals **[MP PET 2002]**

(a) $-\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ (b) $\frac{1}{\sqrt{2}}\tan\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ (c) $\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$ (d) $-\frac{1}{\sqrt{2}}\cot\left(\frac{x}{2} + \frac{\pi}{8}\right) + c$

11. $\int \frac{a\,dx}{b + ce^x} =$ **[MP PET 1988; BIT Ranchi 1979]**

(a) $\frac{a}{b}\log\left(\frac{e^x}{b+ce^x}\right) + c$ (b) $\frac{a}{b}\log\left(\frac{b+ce^x}{e^x}\right) + c$ (c) $\frac{b}{a}\log\left(\frac{e^x}{b+ce^x}\right) + c$ (d) $\frac{b}{a}\log\left(\frac{b+ce^x}{e^x}\right) + c$

12. $\int \sin\sqrt{x}\, dx =$ **[Roorkee 1977]**

(a) $2[\sin\sqrt{x} - \cos\sqrt{x}] + c$ (b) $2[\sin\sqrt{x} - \sqrt{x}\cos\sqrt{x}] + c$
(c) $2[\sin\sqrt{x} + \cos\sqrt{x}] + c$ (d) $2[\sin\sqrt{x} + \sqrt{x}\cos\sqrt{x}] + c$

13. $\int \frac{x^2}{(9-x^2)^{3/2}} dx =$

(a) $\frac{x}{\sqrt{9-x^2}} - \sin^{-1}\frac{x}{3} + c$ (b) $\frac{x}{\sqrt{9-x^2}} + \sin^{-1}\frac{x}{3} + c$ (c) $\sin^{-1}\frac{x}{3} - \frac{x}{\sqrt{9-x^2}} + c$ (d) None of these

14. $\int x\sqrt{\frac{1-x^2}{1+x^2}}\, dx =$

(a) $\frac{1}{2}[\sin^{-1} x^2 + \sqrt{1-x^4}] + c$ (b) $\frac{1}{2}[\sin^{-1} x^2 + \sqrt{1-x^2}] + c$

(c) $\sin^{-1} x^2 + \sqrt{1-x^4} + c$ (d) $\sin^{-1} x^2 + \sqrt{1-x^2} + c$

15. If $\int f(x)\sin x\cos x\,dx = \frac{1}{2(b^2-a^2)}\log(f(x))+c$, then $f(x) =$

(a) $\frac{1}{a^2\sin^2 x + b^2\cos^2 x}$ (b) $\frac{1}{a^2\sin^2 x - b^2\cos^2 x}$ (c) $\frac{1}{a^2\cos^2 x + b^2\sin^2 x}$ (d) $\frac{1}{a^2\cos^2 x - b^2\sin^2 x}$

16. $\int \frac{dx}{4\sin^2 x + 5\cos^2 x} =$ **[AISSE 1986]**

(a) $\frac{1}{\sqrt5}\tan^{-1}\left(\frac{2\tan x}{\sqrt5}\right)+c$ (b) $\frac{1}{\sqrt5}\tan^{-1}\left(\frac{\tan x}{\sqrt5}\right)+c$ (c) $\frac{1}{2\sqrt5}\tan^{-1}\left(\frac{2\tan x}{\sqrt5}\right)+c$ (d) None of these

17. $\int \frac{x^2+1}{x^4-x^2+1}\,dx =$ **[MP PET 1991]**

(a) $\tan^{-1}\left(\frac{1+x^2}{x}\right)+c$ (b) $\cot^{-1}\left(\frac{1+x^2}{x}\right)+c$ (c) $\tan^{-1}\left(\frac{x^2-1}{x}\right)+c$ (d) $\cot^{-1}\left(\frac{x^2-1}{x}\right)+c$

18. $\int (\log x)^2\,dx =$ **[IIT 1971, 77]**

(a) $x(\log x)^2 - 2x\log x - 2x + c$ (b) $x(\log x)^2 - 2x\log x - x + c$

(c) $x(\log x)^2 - 2x\log x + 2x + c$ (d) $x(\log x)^2 - 2x\log x + x + c$

19. The value of $\int \frac{\sqrt{(x^2-a^2)}}{x}\,dx$ will be **[UPSEAT 1999]**

(a) $\sqrt{(x^2-a^2)} - a\tan^{-1}\left[\frac{\sqrt{(x^2-a^2)}}{a}\right]$ (b) $\sqrt{(x^2-a^2)} + a\tan^{-1}\left[\frac{\sqrt{(x^2-a^2)}}{a}\right]$

(c) $\sqrt{(x^2-a^2)} + a^2\tan^{-1}[\sqrt{x^2-a^2}]$ (d) $\tan^{-1} x/a + c$

20. $\int \tan^3 2x\sec 2x\,dx =$ **[IIT 1977]**

(a) $\frac{1}{6}\sec^3 2x - \frac{1}{2}\sec 2x + c$ (b) $\frac{1}{6}\sec^3 2x + \frac{1}{2}\sec 2x + c$

(c) $\frac{1}{9}\sec^2 2x - \frac{1}{3}\sec 2x + c$ (d) None of these

21. $\int x\sin^{-1} x\,dx =$ **[MP PET 1991]**

(a) $\left(\frac{x^2}{2}-\frac{1}{4}\right)\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2}+c$ (b) $\left(\frac{x^2}{2}+\frac{1}{4}\right)\sin^{-1} x + \frac{x}{4}\sqrt{1-x^2}+c$

(c) $\left(\frac{x^2}{2}-\frac{1}{4}\right)\sin^{-1} x - \frac{x}{4}\sqrt{1-x^2}+c$ (d) $\left(\frac{x^2}{2}+\frac{1}{4}\right)\sin^{-1} x - \frac{x}{4}\sqrt{1-x^2}+c$

22. $\int \sqrt{\frac{a-x}{x}}\,dx =$

(a) $a\left[\sin^{-1}\sqrt{\frac{x}{a}} + \sqrt{\frac{x}{a}}\sqrt{\frac{a-x}{a}}\right]+c$ (b) $\sin^{-1}\frac{x}{a} + \frac{x}{a}\sqrt{a^2-x^2}+c$

(c) $a\left[\sin^{-1}\frac{x}{a} - \frac{x}{a}\sqrt{a^2-x^2}\right]+c$ (d) $\sin^{-1}\frac{x}{a} - \frac{x}{a}\sqrt{a^2-x^2}+c$

23. If $x \in \left(\frac{\pi}{4},\frac{3\pi}{4}\right)$, then $\int \frac{\sin x - \cos x}{\sqrt{1-\sin 2x}}e^{\sin x}\cos x\,dx =$

(a) $e^{\sin x}+c$ (b) $e^{\sin x - \cos x}+c$

(c) $e^{\sin x + \cos x}+c$ (d) $e^{\cos x - \sin x}+c$

24. If $\int \frac{4e^x+6e^{-x}}{9e^x-4e^{-x}}\,dx = Ax + B\log(9e^{2x}-4)+C$, then A, B and C are **[IIT 1990]**

(a) $A=\frac{3}{2}, B=\frac{36}{35}, C=\frac{3}{2}\log 3 +$ constant

(b) $A=\frac{3}{2}, B=\frac{35}{36}, C=\frac{3}{2}\log 3 +$ constant

(c) $A=-\frac{3}{2}, B=-\frac{35}{36}, C=-\frac{3}{2}\log 3+\text{constant}$

(d) None of these

25. The value of $\int \sec^3 x\, dx$ will be **[UPSEAT 1999]**

(a) $\frac{1}{2}[\sec x\tan x+\log(\sec x+\tan x)]$

(b) $\frac{1}{3}[\sec x\tan x+\log(\sec x+\tan x)]$

(c) $\frac{1}{4}[\sec x\tan x+\log(\sec x+\tan x)]$

(d) $\frac{1}{8}[\sec x\tan x+\log(\sec x+\tan x)]$

26. $\int \frac{x-1}{(x+1)^3}e^x\, dx=$ **[IIT 1983; MP PET 1990]**

(a) $\frac{-e^x}{(x+1)^2}+c$ (b) $\frac{e^x}{(x+1)^2}+c$

(c) $\frac{e^x}{(x+1)^3}+c$ (d) $\frac{-e^x}{(x+1)^3}+c$

27. If $I=\int e^x\sin 2x\, dx$, then for what value of K, $KI=e^x(\sin 2x-2\cos 2x)+\text{constant}$ **[MP PET 1992]**

(a) 1 (b) 3
(c) 5 (d) 7

28. The value of $\int \frac{dx}{3-2x-x^2}$ will be **[UPSEAT 1999]**

(a) $\frac{1}{4}\log\left(\frac{3+x}{1-x}\right)$ (b) $\frac{1}{3}\log\left(\frac{3+x}{1-x}\right)$

(c) $\frac{1}{2}\log\left(\frac{3+x}{1-x}\right)$ (d) $\log\left(\frac{1-x}{3+x}\right)$

29. $\int x\sqrt{2x+3}\, dx=$ **[AISSE 1985]**

(a) $\frac{x}{3}(2x+3)^{3/2}-\frac{1}{15}(2x+3)^{5/2}+c$

(b) $\frac{x}{3}(2x+3)^{3/2}+\frac{1}{15}(2x+3)^{5/2}+c$

(c) $\frac{x}{2}(2x+3)^{3/2}+\frac{1}{6}(2x+3)^{5/2}+c$

(d) None of these

30. $\int \cos 2\theta\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)d\theta=$ **[IIT 1994]**

(a) $(\cos\theta-\sin\theta)^2\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$

(b) $(\cos\theta+\sin\theta)^2\log\left(\frac{\cos\theta+\sin\theta}{\cos\theta-\sin\theta}\right)$

(c) $\frac{(\cos\theta-\sin\theta)^2}{2}\log\left(\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}\right)$

(d) $\frac{1}{2}\sin 2\theta\log\tan\left(\frac{\pi}{4}+\theta\right)-\frac{1}{2}\log\sec 2\theta$

31. $\int \frac{x^2}{(x\sin x+\cos x)^2}dx=$ **[MNR 1989; RPET 2000]**

(a) $\frac{\sin x+\cos x}{x\sin x+\cos x}$ (b) $\frac{x\sin x-\cos x}{x\sin x+\cos x}$

(c) $\frac{\sin x-x\cos x}{x\sin x+\cos x}$ (d) None of these

32. If $u = \int e^{ax} \cos bx \, dx$ and $v = \int e^{ax} \sin bx \, dx$, then $(a^2 + b^2)(u^2 + v^2) =$

(a) $2e^{ax}$ (b) $(a^2 + b^2)e^{2ax}$
(c) e^{2ax} (d) $(a^2 - b^2)e^{2ax}$

33. If $I_n = \int (\log x)^n \, dx$, then $I_n + nI_{n-1} =$ **[Karnataka CET 2003]**

(a) $x(\log x)^n$ (b) $(x \log x)^n$
(c) $(\log x)^{n-1}$ (d) $n(\log x)^n$

34. $\int e^{x/2} \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) dx =$ **[Roorkee 1982]**

(a) $e^{x/2} \cos\frac{x}{2} + c$ (b) $\sqrt{2}e^{x/2} \cos\frac{x}{2} + c$
(c) $e^{x/2} \sin\frac{x}{2} + c$ (d) $\sqrt{2}e^{x/2} \sin\frac{x}{2} + c$

35. If $\int \frac{2x+3}{x^2 - 5x + 6} dx = 9\ln(x-3) - 7\ln(x-2) + A$, then $A =$ **[MP PET 1992]**

(a) $5\ln(x-2) +$ constant (b) $-4\ln(x-3) +$ constant
(c) Constant (d) None of these

36. $\int \frac{dx}{2 + \cos x} =$

(a) $2\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right) + c$ (b) $\frac{2}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right) + c$
(c) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{1}{\sqrt{3}}\tan\frac{x}{2}\right) + c$ (d) None of these

37. $\int \frac{x}{x^4 + x^2 + 1} dx$ equal to **[MP PET 2004]**

(a) $\frac{1}{3}\tan^{-1}\left(\frac{2x^2+1}{3}\right)$ (b) $\frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{2x^2+1}{\sqrt{3}}\right)$
(c) $\frac{1}{\sqrt{3}}\tan^{-1}(2x^2+1)$ (d) None of these

38. $\int \frac{dx}{(\sin x + \sin 2x)} =$ **[IIT 1984]**

(a) $\frac{1}{6}\log(1-\cos x) + \frac{1}{2}\log(1+\cos x) - \frac{2}{3}\log(1+2\cos x)$

(b) $6\log(1-\cos x) + 2\log(1+\cos x) - \frac{2}{3}\log(1+2\cos x)$

(c) $6\log(1-\cos x) + \frac{1}{2}\log(1+\cos x) + \frac{2}{3}\log(1+2\cos x)$

(d) None of these

39. If $\int \frac{2x+3}{(x-1)(x^2+1)} dx = \log_e\left\{(x-1)^{\frac{5}{2}}(x^2+1)^a\right\} - \frac{1}{2}\tan^{-1}x + A$,
where A is any arbitrary constant, then the value of 'a' is **[MP PET 1998]**

(a) 5/4 (b) – 5/3
(c) – 5/6 (d) – 5/4

40. If $\int \frac{(2x^2+1)\,dx}{(x^2-4)\,(x^2-1)} = \log\left[\left(\frac{x+1}{x-1}\right)^a \left(\frac{x-2}{x+2}\right)^b\right] + C$, then the values of a and b are respectively **[Roorkee 2000]**

(a) 1/2, 3/4 (b) –1, 3/2
(c) 1, 3/2 (d) –1/2, ¾

(Definite Integral)

1. If I is the greatest of the definite integrals

$I_1 = \int_0^1 e^{-x} \cos^2 x\, dx$, $I_2 = \int_0^1 e^{-x^2} \cos^2 x\, dx$

$I_3 = \int_0^1 e^{-x^2} dx$, $I_4 = \int_0^1 e^{-x^2/2} dx$, then

(a) $I = I_1$ (b) $I = I_2$
(c) $I = I_3$ (d) $I = I_4$

2. Let $f(x)$ be a function satisfying $f'(x) = f(x)$ with $f(0) = 1$ and $g(x)$ be the function satisfying $f(x) + g(x) = x^2$. The value of integral $\int_0^1 f(x)\, g(x)\, dx$ is equal to

[AIEEE 2003; DCE 2005]

(a) $\frac{1}{4}(e-7)$ (b) $\frac{1}{4}(e-2)$
(c) $\frac{1}{2}(e-3)$ (d) None of these

3. If $I_m = \int_1^x (\log x)^m dx$ satisfies the relation $I_m = k - lI_{m-1}$, then

(a) $k = e$ (b) $l = m$
(c) $k = \frac{1}{e}$ (d) None of these

4. Let f be a positive function. Let

$I_1 = \int_{1-k}^{k} x\, f\{x(1-x)\} dx$, $I_2 = \int_{1-k}^{k} f\{x(1-x)\} dx$

when $2k - 1 > 0$. Then I_1 / I_2 is **[IIT 1997 Cancelled]**

(a) 2 (b) k
(c) $1/2$ (d) 1

5. If $\int_0^x f(t)\, dt = x + \int_x^1 t\, f(t)\, dt$, then the value of $f(1)$ is

[IIT 1998; AMU 2005]

(a) 1/2 (b) 0
(c) 1 (d) –1/2

6. $\int_0^1 \frac{x^7}{\sqrt{1-x^4}} dx$ is equal to **[AMU 2000]**

(a) 1 (b) $\frac{1}{3}$
(c) $\frac{2}{3}$ (d) $\frac{\pi}{3}$

7. If n is any integer, then $\int_0^\pi e^{\cos^2 x} \cos^3 (2n+1) x\, dx =$

[IIT 1985; RPET 1995; UPSEAT 2001]

(a) x (b) 1
(c) 0 (d) None of these

8. The value of the definite integral $\int_0^1 \frac{x\, dx}{x^3 + 16}$ lies in the interval $[a, b]$. The smallest such interval is

(a) $\left[0, \frac{1}{17}\right]$ (b) [0, 1]
(c) $\left[0, \frac{1}{27}\right]$ (d) None of these

9. Let a,b,c be non-zero real numbers such that $\int_0^1 (1+\cos^8 x)(ax^2+bx+c)\, dx = \int_0^2 (1+\cos^8 x)(ax^2+bx+c)\, dx$

Then the quadratic equation $ax^2 + bx + c = 0$ has

[IIT 1981; CEE 1993]

(a) No root in (0, 2)
(b) At least one root in (0, 2)
(c) A double root in (0, 2)
(d) None of these

10. If $f(x) = \int_{-1}^{x} |t|\, dt$, $x \geq -1$, then **[MNR 1994]**

(a) f and f' are continous for $x+1>0$
(b) f is continous but f' is not continous for $x+1>0$
(c) f and f' are not continous at $x=0$
(d) f is continous at $x=0$ but f' is not so

11. Let $g(x)=\int_0^x f(t)\,dt$ where $\frac{1}{2}\le f(t)\le 1, t\in[0,1]$ and $0\le f(t)\le\frac{1}{2}$ for $t\in(1,2]$, then **[IIT Screening 2000]**

(a) $-\frac{3}{2}\le g(2)<\frac{1}{2}$ (b) $0\le g(2)<2$
(c) $\frac{3}{2}<g(2)\le\frac{5}{2}$ (d) $2<g(2)<4$

12. The value of $\int_{-\pi}^{\pi}\frac{\cos^2 x}{1+a^x}dx, a>0,$ is

[IIT Screening 2001; AIEEE 2005]

(a) π (b) $a\pi$
(c) $\frac{\pi}{2}$ (d) 2π

13. If $f(x)=\frac{e^x}{1+e^x}$, $I_1=\int_{f(-a)}^{f(a)}xg\{x(1-x)\}dx$, and $I_2=\int_{f(-a)}^{f(a)}g\{x(1-x))\}dx$, then the value of $\frac{I_2}{I_1}$ is

[AIEEE 2004]

(a) 1 (b) –3
(c) –1 (d) 2

14. Let $f:R\to R$ and $g:R\to R$ be continuous functions, then the value of the integral

$\int_{-\pi/2}^{\pi/2}[f(x)+f(-x)]\,[g(x)-g(-x)]dx=$

[IIT 1990; DCE 2000; MP PET 2001]

(a) π (b) 1
(c) -1 (d) 0

15. The numbers P, Q and R for which the function $f(x)=Pe^{2x}+Qe^x+Rx$ satisfies the conditions $f(0)=-1$, $f'(\log 2)=31$ and $\int_0^{\log 4}[f(x)-Rx]dx=\frac{39}{2}$ are given by

(a) $P=2,\ Q=-3,\ R=4$ (b) $P=-5,\ Q=2,\ R=3$
(c) $P=5,\ Q=-2,\ R=3$ (d) $P=5,\ Q=-6,\ R=3$

16. $\left[\sum_{n=1}^{10}\int_{-2n-1}^{2n}\sin^{27}x\,dx\right]+\left[\sum_{n=1}^{10}\int_{2n}^{2n+1}\sin^{27}x\,dx\right]$ equals

[MP PET 2002]

(a) 27^2 (b) -54
(c) 36 (d) 0

17. Let $\int_0^1 f(x)dx=1$, $\int_0^1 x\,f(x)dx=a$ and $\int_0^1 x^2 f(x)\,dx=a^2$, then the value of $\int_0^1(x-a)^2 f(x)dx=$ **[IIT 1990]**

(a) 0 (b) a^2
(c) a^2-1 (d) a^2-2a+2

18. Given that $\int_0^{\infty}\frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)(x^2+c^2)}=\frac{\pi}{2(a+b)(b+c)(c+a)}$, then the value of $\int_0^{\infty}\frac{x^2dx}{(x^2+4)(x^2+9)}$ is

[Karnataka CET 1993]

(a) $\frac{\pi}{60}$ (b) $\frac{\pi}{20}$
(c) $\frac{\pi}{40}$ (d) $\frac{\pi}{80}$

19. If $I(m,n)=\int_0^1 t^m(1+t)^n dt$, then the expression for $I(m,n)$ in terms of $I(m+1,n-1)$ is **[IIT Screening 2003]**

(a) $\frac{2^n}{m+1}-\frac{n}{m+1}I(m+1,n-1)$

(b) $\frac{n}{m+1}I(m+1,n-1)$

(c) $\frac{2^n}{m+1}+\frac{n}{m+1}I(m+1,n-1)$

(d) $\frac{m}{n+1}I(m+1,n-1)$

20. $\lim_{n\to\infty}\frac{1+2^4+3^4+....+n^4}{n^5}-\lim_{n\to\infty}\frac{1+2^3+3^3+....+n^3}{n^5}=$ **[AIEEE 2003]**

(a) $\frac{1}{30}$ (b) Zero

(c) $\frac{1}{4}$ (d) $\frac{1}{5}$

21. If $\int_0^{t^2} xf(x)dx=\frac{2}{5}t^5$, $t>0$, then $f\left(\frac{4}{25}\right)=$ **[IIT Screening 2004]**

(a) $\frac{2}{5}$ (b) $\frac{5}{2}$

(c) $-\frac{2}{5}$ (d) None of these

22. For which of the following values of m, the area of the region bounded by the curve $y=x-x^2$ and the line $y=mx$ equals $\frac{9}{2}$ **[IIT 1999]**

(a) –4 (b) –2

(c) 2 (d) 4

23. Area enclosed between the curve $y^2(2a-x)=x^3$ and line $x=2a$ above x-axis is **[MP PET 2001]**

(a) πa^2 (b) $\frac{3\pi a^2}{2}$

(c) $2\pi a^2$ (d) $3\pi a^2$

24. What is the area bounded by the curves $x^2+y^2=9$ and $y^2=8x$ is **[DCE 1999]**

(a) 0 (b) $\frac{2\sqrt{2}}{3}+\frac{9\pi}{2}-9\sin^{-1}\left(\frac{1}{3}\right)$

(c) 16π (d) None of these

25. The area bounded by the curves $y=|x|-1$ and $y=-|x|+1$ is **[IIT Screening 2002]**

(a) 1 (b) 2

(c) $2\sqrt{2}$ (d) 4

26. The volume of spherical cap of height h cut off from a sphere of radius a is equal to **[UPSEAT 2004]**

(a) $\frac{\pi}{3}h^2(3a-h)$ (b) $\pi(a-h)(2a^2-h^2-ah)$

(c) $\frac{4\pi}{3}h^3$ (d) None of these

27. If for a real number y, $[y]$ is the greatest integer less than or equal to y, then the value of the integral $\int_{\pi/2}^{3\pi/2}[2\sin x]dx$ is **[IIT 1999]**

(a) $-\pi$ (b) 0

(c) $-\frac{\pi}{2}$ (d) $\frac{\pi}{2}$

28. If $f(x)=A\sin\left(\frac{\pi x}{2}\right)+B$, $f'\left(\frac{1}{2}\right)=\sqrt{2}$ and $\int_0^1 f(x)dx=\frac{2A}{\pi}$, then the constants A and B are respectively **[IIT 1995]**

(a) $\frac{\pi}{2}$ and $\frac{\pi}{2}$ (b) $\frac{2}{\pi}$ and $\frac{3}{\pi}$

(c) $\frac{4}{\pi}$ and 0 (d) 0 and $-\frac{4}{\pi}$

29. If $I_n=\int_0^{\infty}e^{-x}x^{n-1}dx$, then $\int_0^{\infty}e^{-\lambda x}x^{n-1}dx=$

(a) λI_n (b) $\frac{1}{\lambda}I_n$

(c) $\frac{I_n}{\lambda^n}$ (d) $\lambda^n I_n$

30. $I_n = \int_0^{\pi/4} \tan^n x\, dx$, then $\lim_{n\to\infty} n[I_n + I_{n-2}]$ equals **[AIEEE 2002]**

(a) 1/2 (b) 1
(c) ∞ (d) 0

31. The area bounded by the curves $y = \ln x$, $y = \ln|x|$, $y = |\ln x|$ and $y = |\ln|x||$ is **[AIEEE 2002]**

(a) 4 *sq. unit* (b) 6 *sq. unit*
(c) 10 *sq. unit* (d) None of these

32. $\int_0^{\pi} \frac{\sin\left(n+\frac{1}{2}\right)x}{\sin x} dx$, $(n \in N)$ equals **[Kurukshetra CEE 1998]**

(a) $n\pi$ (b) $(2n+1)\frac{\pi}{2}$
(c) π (d) 0

33. If $\int_0^1 e^{x^2}(x-\alpha)\, dx = 0$, then **[MNR 1994; Pb. CET 2001; UPSEAT 2000]**

(a) $1 < \alpha < 2$ (b) $\alpha < 0$
(c) $0 < \alpha < 1$ (d) None of these

34. $\int_{\pi}^{10\pi} |\sin x|\, dx$ is **[AIEEE 2002]**

(a) 20 (b) 8
(c) 10 (d) 18

35. $\int_{-\pi}^{\pi} \frac{2x(1+\sin x)}{1+\cos^2 x} dx$ is **[AIEEE 2002]**

(a) $\pi^2/4$ (b) π^2
(c) 0 (d) $\pi/2$

36. On the interval $\left[\frac{5\pi}{3}, \frac{7\pi}{4}\right]$, the greatest value of the function $f(x) = \int_{5\pi/3}^{x} (6\cos t - 2\sin t)\, dt =$

(a) $3\sqrt{3} + 2\sqrt{2} + 1$ (b) $3\sqrt{3} - 2\sqrt{2} - 1$
(c) Does not exist (d) None of these

37. If $I_1 = \int_0^1 2^{x^2} dx$, $I_2 = \int_0^1 2^{x^3} dx$, $I_3 = \int_1^2 2^{x^2} dx$, $I_4 = \int_1^2 2^{x^3} dx$, then **[AIEEE 2005]**

(a) $I_3 = I_4$ (b) $I_3 > I_4$
(c) $I_2 > I_1$ (d) $I_1 > I_2$

38. If $2f(x) - 3f\left(\frac{1}{x}\right) = x$, then $\int_1^2 f(x)\, dx$ is equal to **[J & K 2005]**

(a) $\frac{3}{5}\ln 2$ (b) $\frac{-3}{5}(1+\ln 2)$
(c) $\frac{-3}{5}\ln 2$ (d) None of these

39. If $\int_a^b x^3 dx = 0$ and $\int_a^b x^2 dx = \frac{2}{3}$, then the value of a and b will be respectively **[AMU 2005]**

(a) 1, 1 (b) $-1, -1$
(c) $1, -1$ (d) $-1, 1$

40. The sine and cosine curves intersects infinitely many times giving bounded regions of equal areas. The area of one of such region is **[DCE 2005]**

(a) $\sqrt{2}$ (b) $2\sqrt{2}$
(c) $3\sqrt{2}$ (d) $4\sqrt{2}$